

## Broadening of the surface Andreev bound states band of superfluid $^3\text{He-B}$ on a partially specular wall

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A measurement of the surface Andreev bound states of the superfluid  $^3\text{He-B}$  phase under different boundary conditions is presented. Transverse-acoustic impedance spectroscopy was used to determine the bandwidth of the bound states on a wall with various specularities. It was found that the band was broader for larger specularities and filled up the superfluid gap above a critical specularities. Specularity was controlled by coating the wall with thin layers of  $^4\text{He}$  and was evaluated separately using acoustic impedance measurement in the normal-fluid phase.

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### I. INTRODUCTION

For unconventional superfluids and superconductors of nonzero angular-momentum pairing, the order parameters change their sign depending on the direction of momentum. This sign change leads to a strong suppression of the order parameter in the vicinity of a surface due to destructive interference. At the same time, low-lying quasiparticle states appear in the region, which are referred to as surface Andreev bound states (SABSs). These states are manifested as a zero-energy peak in the surface density of states (SDOS) of unconventional superconductors measured by tunneling spectroscopy and have been an important research field in surface states of superconductors.<sup>1-5</sup> The formation of SABS is quite a global phenomenon in numerous superconductor systems. In the  $d$ -wave superconductors, some latest essential developments were reported: the first observation of SABS in electron-doped cuprate bicrystal junctions<sup>6</sup> and hybrid conventional-unconventional junctions Nb-YBCO.<sup>7</sup> In the  $p$ -wave superconductors, SABSs were also shown to exist experimentally.<sup>8-10</sup>

Although symmetry of the bulk  $p$ -wave superfluid  $^3\text{He}$  has been established more firmly than any  $p$ -wave superconductors, the existence of SABS in superfluid  $^3\text{He}$  was only recently shown by transverse-acoustic impedance spectroscopy.<sup>11,12</sup> A strong effect of the surface boundary condition on the SABS in the  $B$  phase of superfluid  $^3\text{He}$  has been found. The bandwidth of SABS has been systematically investigated using impedance measurements with control of the boundary condition by coating the wall with thin layers of  $^4\text{He}$ .

The study of SABS of superfluid  $^3\text{He}$  is intriguing because  $^3\text{He}$  has several advantages over other unconventional superconductors.  $^3\text{He}$  is free of impurities and defects and is isotropic because it is a very low-temperature liquid without any underlying lattice. Its bulk material parameters have been extensively investigated experimentally not to have suffered from these imperfections. This has allowed a quantitative and theoretically firm understanding of various physical phenomena,<sup>13,14</sup> including SABS. It is well known that for the case of superconductors observation of a zero-bias conductance peak in the conductance measurements does not necessarily prove the unconventional nature of the supercon-

ductor investigated as there are many other possible mechanisms that may generate the peak. Superfluid  $^3\text{He}$  is a well-established  $p$ -wave Bardeen-Cooper-Schrieffer (BCS) state and there is no doubt about the unconventionality.

Bulk parameters, such as the superfluid transition temperature  $T_c$  or the superfluid gap energy  $\Delta$ , and the stability of the well-established pairing states can be tuned by changing the pressure. The boundary conditions of a wall can be altered *in situ* by coating the wall with a few atomic layers of  $^4\text{He}$ .<sup>15-17</sup>  $^4\text{He}$  is absorbed on a wall exclusively due to its larger mass and smaller zero-point fluctuation than  $^3\text{He}$ . This controllability of the physical parameters and the experimental conditions, which is rarely realized for any superconductors, has enabled the systematic study of SABS.

We have studied the  $B$  phase of superfluid  $^3\text{He}$ , which is a realization of the Balian-Werthamer (BW) state in  $p$ -wave BCS systems. The BW state is the pseudoisotropic state having a uniform energy gap among the  $p$ -wave pairing states.<sup>14</sup> The BW state of  $p$ -wave superconductors is almost impossible to realize because lattice symmetry usually stabilizes more anisotropic states. Therefore, using superfluid  $^3\text{He}$  is presumably the only way to investigate SABS of the BW state.

Superfluid  $^3\text{He}$  in the vicinity of a wall has been studied theoretically.<sup>18-26</sup> Calculations of the SDOS for the BW state on walls with various specularities<sup>24,25</sup> are shown in Fig. 1(a).  $S$  is the specular parameter of the wall.  $S=1$  (0) corresponds to the specular (diffusive) limit. For a partially specular wall ( $0 < S < 1$ ), a fraction  $S$  of quasiparticles is specularly scattered, conserving the parallel component of the momentum. The remaining  $1-S$  quasiparticles are diffusely scattered. In the limit of diffusive scattering  $S=0$ , there appears a SABS band centered at the Fermi energy. A peculiar feature of the SDOS is the very sharp edge of the band at  $\Delta^*$ . With increasing  $S$ , the spectral weight around the Fermi energy is shifted toward the band edge and  $\Delta^*$  becomes larger. This broadening of the band leads to gapless SDOS as  $S$  increases further. Experimentally, however, the SDOS on a partially specular wall has never been reported.

### II. EXPERIMENTS

A sample cell made of oxygen-free high-conductivity copper was installed in a nuclear demagnetization refrigera-

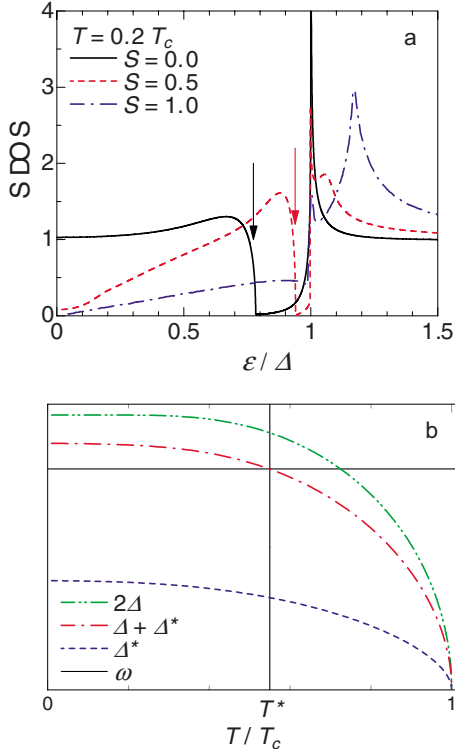


FIG. 1. (Color online) (a) Theoretical calculations of the SDOS for the BW state by Nagato *et al.* (Refs. 24 and 25). The vertical axis is normalized to the normal-state value. In the diffusive limit  $S=0$ , a SABS band with a clear edge at  $\Delta^*$ , represented by the black (left) arrow, appears within the bulk energy gap  $\Delta$ . As  $S$  increases,  $\Delta^*$  becomes larger (the red (right) arrow) and eventually merges into a quasiparticle continuum above  $\Delta$ . (b) Temperature dependence of the characteristic energies. The vertical line is  $T^*$  at which the condition  $\hbar\omega = \Delta(T^*) + \Delta^*(T^*)$  is met.

tor precooled using a  $^3\text{He}$ - $^4\text{He}$  dilution refrigerator. A vibrating-wire thermometer, which was used to directly measure the liquid temperature, was installed in the sample cell, in addition to a  $^3\text{He}$  melting curve thermometer mounted on the nuclear stage. To coat the wall with  $^4\text{He}$  layers, a controlled amount of  $^4\text{He}$  was first introduced into the sample cell at 10 K, and it was kept overnight to achieve equilibrium.  $^3\text{He}$  was then condensed into the cell below 0.3 K. The thickness of the  $^4\text{He}$  coverage can be easily determined due to the large surface area of the sintered-silver heat exchanger ( $142\text{ m}^2$ ) in the cell. For  $^4\text{He}$  coverage of  $n=2.7$  and 3.6 layers, the amount of  $^4\text{He}$  is 40.1 and 51.2 mmol/m $^2$ , respectively.<sup>27</sup>

Transverse-acoustic impedance is a complex quantity defined by the ratio of the velocity of the transversely oscillating wall  $u$  and the shear stress of the liquid acting on the wall  $\Pi$  as  $Z = Z' + iZ'' = \Pi/u$ . Real and imaginary components,  $Z'$  and  $Z''$ , can be obtained from the quality factor  $Q$  and the resonance frequency  $f$  of ac-cut quartz transducers immersed in the liquid  $^3\text{He}$  by

$$Z' = \frac{1}{4} m \pi Z_q \Delta (1/Q) \quad (1)$$

and

$$Z'' = \frac{1}{2} m \pi Z_q \Delta f / f, \quad (2)$$

where  $Z_q$  is the acoustic impedance of quartz and  $m$  is the harmonics number of the transducer.<sup>13</sup>  $\Delta f$  and  $\Delta(1/Q)$  represent the changes from the unloaded condition. Two ac-cut quartz transducers with coaxial electrodes were used; their fundamental frequencies were 9.56 and 15.5 MHz. The resonance shape of the transducers was measured using a cw bridge at their fundamental frequencies and at several harmonics. The resonances were fitted using a Lorentzian curve to obtain  $Q$  and  $f$ . We chose the excitation voltage to be low enough to work in the linear regime. Temperature dependence of  $Z$  did not depend on the excitations both in the superfluid and normal-fluid phases, and thus a nonlinear response and a heating problem should be negligible.

The temperature dependence of the transverse-acoustic impedance revealed the characteristic subgap structure in the diffusive limit in Refs. 11 and 12 for a bare surface without  $^4\text{He}$  coating. A clear kink in  $Z'$  and a peak in  $Z''$  appear at a particular temperature  $T^*$ , which is dependent on the measurement frequency  $f$  and the pressure  $P$ . The temperature dependence  $Z(T)$  is well reproduced theoretically,<sup>25</sup> and it was found that the kink and peak are weak singularities appearing at  $T^*$ , at which the condition

$$\hbar\omega = \Delta(T^*) + \Delta^*(T^*) \quad (3)$$

is met, where  $\omega = 2\pi f$ . The temperature dependence of the characteristic energies is shown in Fig. 1(b). This was made use to obtain  $\Delta^*(T^*)$  using the  $\Delta(T)$  given by the weak-coupling-plus model.<sup>13</sup>  $\Delta^*$  data were collected under various conditions in a zero magnetic field, with and without  $^4\text{He}$  coating, and at various  $f$  and  $P$ .

### III. RESULTS AND DISCUSSION

Typical behavior of the measured temperature dependence of  $Z(T)$  is shown in Fig. 2 at  $f=47.8$  MHz and  $P=10$  bar. Changes from the normal-state value  $Z_0$  are plotted. Black circles indicate the case for the bare wall, red triangles indicate that for coating with  $n=2.7$  layers of  $^4\text{He}$ , and blue squares indicate  $n=3.6$  layers. Coating of the wall has two effects on  $Z$ ; one is the reduction in overall temperature dependence, and the other is the shift of characteristic features to higher temperature. The reduction in  $Z(T)$  is a clear indication that  $S$  is successfully increased by coating the wall with  $^4\text{He}$  layers. Specular scattering conserves the momentum of quasiparticles parallel to the wall, and thus such quasiparticles do not contribute to  $Z$ .

The shift of the characteristic temperatures is very important because it is a clear evidence that the SDOS is modified by the coating and that the  $Z(T)$  measurement does pick up information regarding SABS. The coating modifies only the states in the vicinity of the wall within the order of the coherence length, while the bulk of the superfluid remains unchanged. If the  $Z(T)$  measurement had sensed the bulk property, no shift in the temperature dependence would have been seen. The vertical lines in Fig. 2 indicate  $T^*$  for each measurement. The shift of  $T^*$  to higher temperature is the result

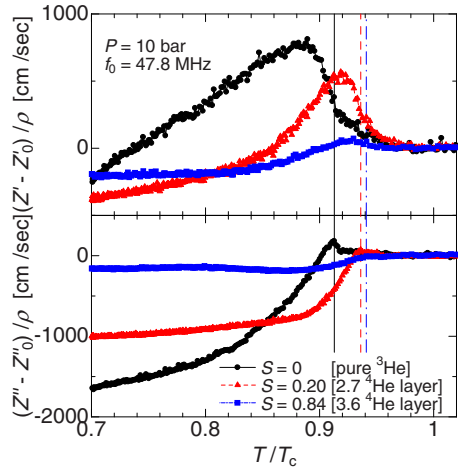


FIG. 2. (Color online) Temperature dependence of the real  $Z'$  and the imaginary component  $Z''$  of the transverse-acoustic impedance with and without  $^4\text{He}$  coatings. Changes from the normal-state value  $Z_0$  are shown at  $P=10$  bar and  $f=47.8$  MHz. Vertical lines indicate  $T^*$  that shift to higher temperature for thicker coatings of  $^4\text{He}$ .

of the broadening of the SABS band by the coating. The condition given by Eq. (3) is met at a higher temperature than in the bare wall case if  $\Delta^*$  is larger for a more thickly coated wall. This is an experimental confirmation of the sensitivity of SABS to the boundary condition in the  $B$  phase of superfluid  $^3\text{He}$ .

In order to estimate the value of  $S$ ,  $Z(T)$  was measured in the normal phase at all  $f$ ,  $P$ , and  $n$ , where measurements were conducted in the superfluid phase. While  $Z''$  suffered from a large temperature-dependent background of the transducer itself in the normal-fluid region,  $Z'$  had much smaller background and its temperature dependence was dominated by the  $^3\text{He}$  property.<sup>28</sup>  $Z'(T)$  was adopted for the determination of  $S$ . Figure 3(a) shows an example of  $Z'(T)$  in the normal-fluid phase, where the black circles, red triangles, and blue squares represent  $n=0$ , 2.7, and 3.6 layers, respectively, of  $^4\text{He}$  at  $P=17$  bar and  $f=15.5$  MHz. The fitting procedure by Milliken *et al.*<sup>29</sup> was followed. The temperature dependence of  $Z(T)$  in normal-fluid  $^3\text{He}$  is described by the Fermi-liquid theory<sup>30,31</sup> and given in the diffusive limit ( $S=0$ ) as

$$\frac{Z(T, S=0)}{\rho v_F} = \frac{3 + F_1^S}{8(1 + \beta^{1/2})} \quad (4)$$

and

$$\beta = \frac{1}{4} \left( 3 + F_1^S - \frac{i\omega\tau}{1 - i\omega\tau} \right) \left( \frac{1}{1 + F_2^S/5} - \frac{\xi_2}{i\omega\tau} \right), \quad (5)$$

where  $\rho$  and  $v_F$  are the density of liquid  $^3\text{He}$  and the Fermi velocity, respectively, and  $F_1^S$  and  $F_2^S$  are the Fermi-liquid parameters,  $\tau$  is the quasiparticle relaxation time proportional to  $T^{-2}$ ,  $\xi_2 = \tau/\tau_2 = 0.35$ , and  $\tau_2$  is the viscous relaxation time.  $Z'(T)$  was fitted for  $n=0$ , as shown in Fig. 3(a). The  $S$  dependence of  $Z(T, S)$  is given as

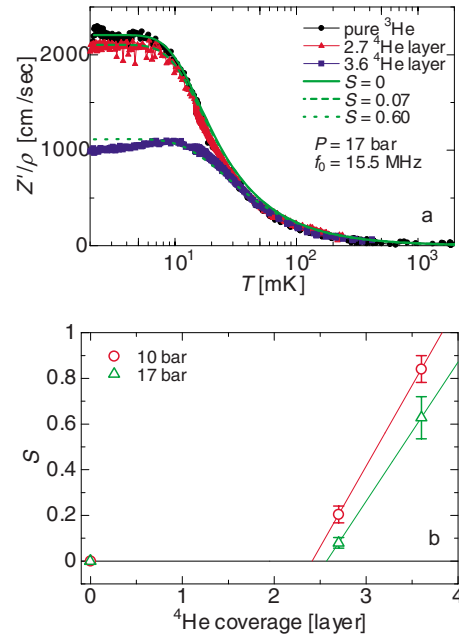


FIG. 3. (Color online) (a) Real component of the transverse-acoustic impedance in the normal-fluid phase at  $P=17$  bar and  $f=15.5$  MHz. Closed symbols represent the experimental data and the lines are the fittings according to the Fermi-liquid theory for the determination of  $S$ . (b)  $S$  of the wall as a function of  $^4\text{He}$  coverage at  $P=10$  and 17 bar.

$$\frac{Z(T, S)}{Z(T, S=0)} = \frac{1 - S}{1 - S + SZ(T, S=0)/L_1(0)}, \quad (6)$$

where  $L_1(0) = 15\eta/16\lambda$ , and  $\eta$  and  $\lambda$  are the viscosity and the quasiparticle mean-free path, respectively.  $Z'(T)$  was fitted using  $S$  for the  $^4\text{He}$  coating. Other parameters used are the same as in the  $n=0$  case. We obtained  $S=0$ , 0.07, and 0.60. This fitting reproduces the experiment well, as seen in Fig. 3(a), except for a small anomalous decrease in the case of  $n=3.6$  at low temperature. This anomalous decrease is more pronounced for larger  $n$  and at higher  $f$ , and it is the cause of the error in estimation of  $S$ .  $Z(T)$  is known to have a small frequency dependence that Eqs. (4) and (5) cannot account for.<sup>32</sup> We also observed this behavior that results in the scatter in  $F_2^S$ ; this scatter is another cause of the error in  $S$ .

Although there is a slightly larger uncertainty in  $S$  when  $Z'(T)$  shows the anomaly,  $S$  can nonetheless be estimated with reasonable accuracy for our purposes. Figure 3(b) shows the estimation of  $S$  for  $n=0$ , 2.7, and 3.6 layers at  $P=10$  and 17 bar.  $S$  is larger for larger  $n$  and smaller at higher  $P$ . The increase in  $S$  is believed to be a result of the superfluidity of the  $^4\text{He}$  layers. It is known from the torsional oscillator measurements and the fourth sound measurement that the layers below  $n \sim 2$  are inert and do not change  $S$ .<sup>15–17</sup> The reduction in  $S$  at high pressure has also been observed and is regarded as the solidification of the  $^4\text{He}$  layers.

The temperature dependence of  $\Delta^*$  is shown in Fig. 4, where the vertical axis is normalized to the bulk energy gap and the horizontal axis is normalized to the transition temperature. The thick lines are used to indicate the data trends.

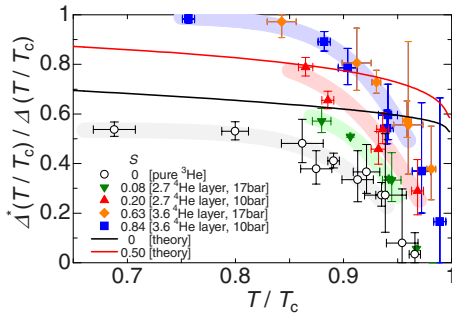


FIG. 4. (Color online) Temperature dependence of  $\Delta^*$  normalized to the bulk gap  $\Delta$  for various  $S$ . Thick lines are used to indicate trends, and the thin lines represent theoretical calculations for  $S = 0$  and  $0.5$ .

Broadening of the SABS band is shown in this figure.  $\Delta^*/\Delta$  becomes larger as  $S$  increases, although the scatter is rather large in the high-temperature region  $T \geq 0.9T_c$ . The large scatter is caused by the steep change in  $\Delta(T/T_c)$  in the region because uncertainty in the temperature results in an error in  $\Delta^*(T/T_c)$ .  $\Delta^*/\Delta$  increases slightly with cooling in the low-temperature region, and the theoretical data also show this tendency. The thin black (lower) and red (upper) lines represent the theoretical calculation for  $S=0$  and  $0.5$ , respectively, and they reproduce a qualitative feature of the temperature and specularity dependence. Although the theory explains  $Z(T)$  well and has semiquantitative agreement, as given in Ref. 25, it fails to have a quantitative agreement with  $\Delta^*(T/T_c)$  because this requires more elaborate calculations. The theory was formulated for the weak coupling limit, and a strong-coupling correction may help to achieve better agreement.

An isotherm of  $\Delta^*/\Delta$  at  $T=0.88T_c$  is plotted against  $S$  in Fig. 5, and the line is used to indicate the data trend. A steep increase is observed for small  $S$  and saturates at larger  $S$ .  $\Delta^*/\Delta$  is close to unity in the large  $S$  region. It is very likely that the band fills the gap above a critical specularity  $S_c$  and SDOS becomes gapless in this region as the theory predicts. Although an accurate estimation of  $S_c$  is not easy, we have roughly speculated that  $S_c = 0.4 \pm 0.1$  by simply extrapolating the data at small  $S$  linearly. It is hoped that our finding will trigger more theoretical research on this intriguing problem of how a wall, which induces anisotropic scattering of quasiparticles, influences the BW state that has an isotropic superfluid gap even with  $p$ -wave symmetry. The existence of

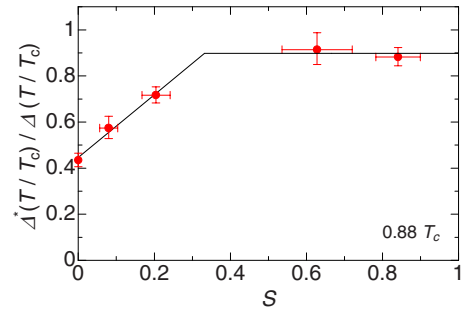


FIG. 5. (Color online) Isotherm of  $\Delta^*$  normalized to the bulk gap  $\Delta$  at  $0.88T_c$  as a function of  $S$ . The line is used to indicate the data trend.  $\Delta^*/\Delta$  is larger for larger  $S$  and saturates close to unity.

SABS was also reported by precise surface specific-heat measurements<sup>33</sup> and transverse wave measurements.<sup>34</sup> It will be informative to measure the surface specific heat and the damping of transverse wave with  $^4\text{He}$  coating in order to see if the modification of SDOS can be detected by the independent experimental methods.

#### IV. SUMMARY

SABS on a wall of various specularities was revealed in the  $B$  phase of superfluid  $^3\text{He}$  using transverse-acoustic impedance spectroscopy. To obtain the result the remarkable features of superfluid  $^3\text{He}$  were fully utilized; the boundary condition of a wall can be controlled *in situ* by coating the wall with a few atomic layers of  $^4\text{He}$ , and specularity can be estimated separately by impedance measurements in the normal-fluid phase. The bandwidth of SABS is broader with larger specularity, and the SDOS is gapless above a critical specularity. This behavior agrees qualitatively with the theoretical predictions. A more elaborate theory is required to establish quantitative agreement, for example, by taking strong-coupling corrections and so on into account.

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